STABILITY ANALYSIS OF THE AXISYMMETRIC MOTION OF A GAS

A. V. Il'yushchenko and S. A. Kholin

UDC 532.5

We consider the problem of the stability of the compression and expansion of an axisymmetrical gas with a spatially constant density. The analogous problem for the spherically symmetrical case has been investigated previously [1].

For the unperturbed motion we adopt the self-similar solution

$$\rho = \rho_0 t_0^2/t^2, \quad u = r/t,$$

where r is the Euler distance to the z axis, ρ_0 and t_0 are constants, $t \to +\infty$ in expansion, and $t \to -\infty$ in compression.

We assume below that the motion is adiabatic, with adiabatic exponent $\gamma > 1$. This unperturbed motion corresponding to cylindrical compression (or expansion) of a gas column with a pressure at the boundary of the gas that varies according to the power law

$$p = A\rho^\gamma = p_0(t_0/t)^{2\gamma}.$$

Qualitatively, instability sets in because the problem involves two velocities: the velocity of the gas $u = r/t$, which depends on the radius, and the velocity of sound, which is constant in space. We represent the perturbed motion in the form

$$\rho = \rho_0 \left(t_0^2/t^2\right)[1 + \omega(r, t)], \; u = (r/t)[1 + v(r, t)].$$

We also regard the perturbation as a small quantity and retain only terms linear in ω and v in the equations. If in the equation of continuity and the Euler equation

$$\partial\rho/\partial t + \operatorname{div} \rho u = 0, \; \partial u/\partial t + (u\nabla)u = -(1/\rho)\nabla p$$

we transform to Lagrangian coordinates t, $R = r/t$, we obtain the following expressions for the density and velocity perturbations:

$$t\partial v/\partial t + v = -(c^2/R)\partial\omega/\partial R, \quad t\partial\omega/\partial t + (1/R)\partial R^2 v/\partial R = 0.$$

Consolidating these equations, we obtain a single equation for $\omega(R, t)$:

$$t^2\partial^2\omega/\partial t^2 + 2t\partial\omega/\partial t - c^2\{\partial^2\omega/\partial R^2 + (1/R)\partial\omega/\partial R\} = 0,$$

the solution of which we expand in a series of Bessel functions:

$$\omega(R, t) = \sum_k \omega(k, t) J_0(kR).$$

The function $\omega(k, t)$ satisfies the equation

$$t^2\partial^2\omega/\partial t^2 + 2t\partial\omega/\partial t + c^2k^2\omega = 0.$$

Since in this equation $c^2 = c_0^2|t|^{-2(\gamma-1)}$, where c_0 is a constant, for $\gamma \neq 1$ its solution has the form $\omega(k, t) = |t|^{-1/2}\{AJ_\nu(x) + BJ_{-\nu}(x)\}$, where $\nu = \frac{1}{2}(\gamma - 1)$; $x = [c_0k/(\gamma-1)]|t|^{-(\gamma-1)}$; and A, B, D are constants.

As $t \to -0$

$$\omega(k, t) \to D|t|^{(1/2)(\gamma-2)} \cos(x + \pi\nu/2 - \pi/4).$$

Hence it is clear that in compression, if $\gamma < 2$, the motion will be unstable. The amplitude of the standing wave grows in an oscillating manner. For $\gamma > 2$ the motion will be stable.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 83-84, May-June, 1982. Original article submitted March 20, 1981.

 0021-8944/82/2303-0386$07.50

In the isothermal case ($\gamma = 1$)$\omega(k, t)$ has a power-law dependence on the time:

$$\omega(k, t) = C_1 t^{\alpha_1} + C_2 t^{\alpha_2},$$

where $\alpha_{1,2} = 1/2 \pm \sqrt{(1/4 - c_0^2 k^2)^{1/2}}$ and C_1, C_2 are constants. When α is complex-valued, $C_1 = \bar{C}_2$. In the isothermal case the motion is unstable, and the amplitude growth depends on the wavelength.

If we express the growth of the perturbations in terms of the relative compression ρ/ρ_0, we have

$$(\Delta\rho/\rho)/(\Delta\rho/\rho)_0 \leqslant (\rho/\rho_0)^{(1/4)(2-\gamma)} < (\rho/\rho_0)^{1/4},$$

because for real gases $1 \leq \gamma \leq 2$.

LITERATURE CITED

1. S. A. Kholin, "Stability analysis of the motion of a compressible gas," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1965).

PROPAGATION OF FINITE-AMPLITUDE PRESSURE PERTURBATIONS IN A BUBBLING VAPOR–LIQUID MEDIUM

V. E. Nakoryakov, B. G. Pokusaev,
N. A. Pribaturin, and I. R. Shreiber

UDC 532.593:532.529

The investigation of the propagation of pressure perturbations in a liquid saturated with vapor bubbles has produced two different models describing this process. In [1] the wave evolution process is analyzed from the point of view of a thermodynamic-equilibrium model, in which the characteristic sound velocity is calculated in the form [2]

$$c_+ = \mu r p_0 / \left(B\rho_1 T_0 \left(c_{p_1} T_0\right)^{1/2}\right),$$

where p and T are the pressure and temperature of the medium, ρ is the density, c_p is the specific heat, r is the latent heat of phase transition, B is the gas constant, and μ is the molecular weight. We use the indices 1 and 2 everywhere to designate the liquid and the vapor respectively, and the index 0 for the unperturbed state. However, it is inferred from experiments [3-5] that the gas dynamics of a vapor–liquid medium with a bubble structure must be formulated on the basis of a nonequilibrium approach. A model has been proposed in [6] for the propagation of pressure disturbances with allowance for the unsteady behavior of the heat and mass transfer at the bubble–liquid phase interface during the transmission of the pressure pulse. As the characteristic velocity in this model we adopt the "frozen" sound velocity c_0, the value of which can be determined from the expression

$$\frac{1}{c_0^2} = \frac{(1-\varphi_0)^2}{c_1^2} + \frac{\varphi_0 (1-\varphi_0)\rho_1}{\gamma p_0},$$

in which φ_0 is the initial vapor content and γ is the adiabatic exponent for the vapor. The experiments reported in [5] show that the model used in [6] for the heat transfer between a vapor bubble and a liquid well describes the dynamics of bubbles for an arbitrary variation of the external conditions (pressure or temperature). The same experiments also show that the behavior of bubbles in a pressure wave is strongly mirrored in the structure and evolution of the waves. It was observed earlier [4] that under definite conditions the evolution of a pressure perturbation in a liquid containing vapor bubbles can be affected not only by interphase heat and mass transfer, but also by nonlinear and dispersion effects, which are typical of a bubbling gas–liquid medium [7].

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 84-90, May-June, 1982. Original article submitted April 6, 1981.